Exploring the Optimization of Water Bottle Rocket Design Using Computational Models

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MAE 250

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Fig. 1. Final Water Bottle Rocket Design

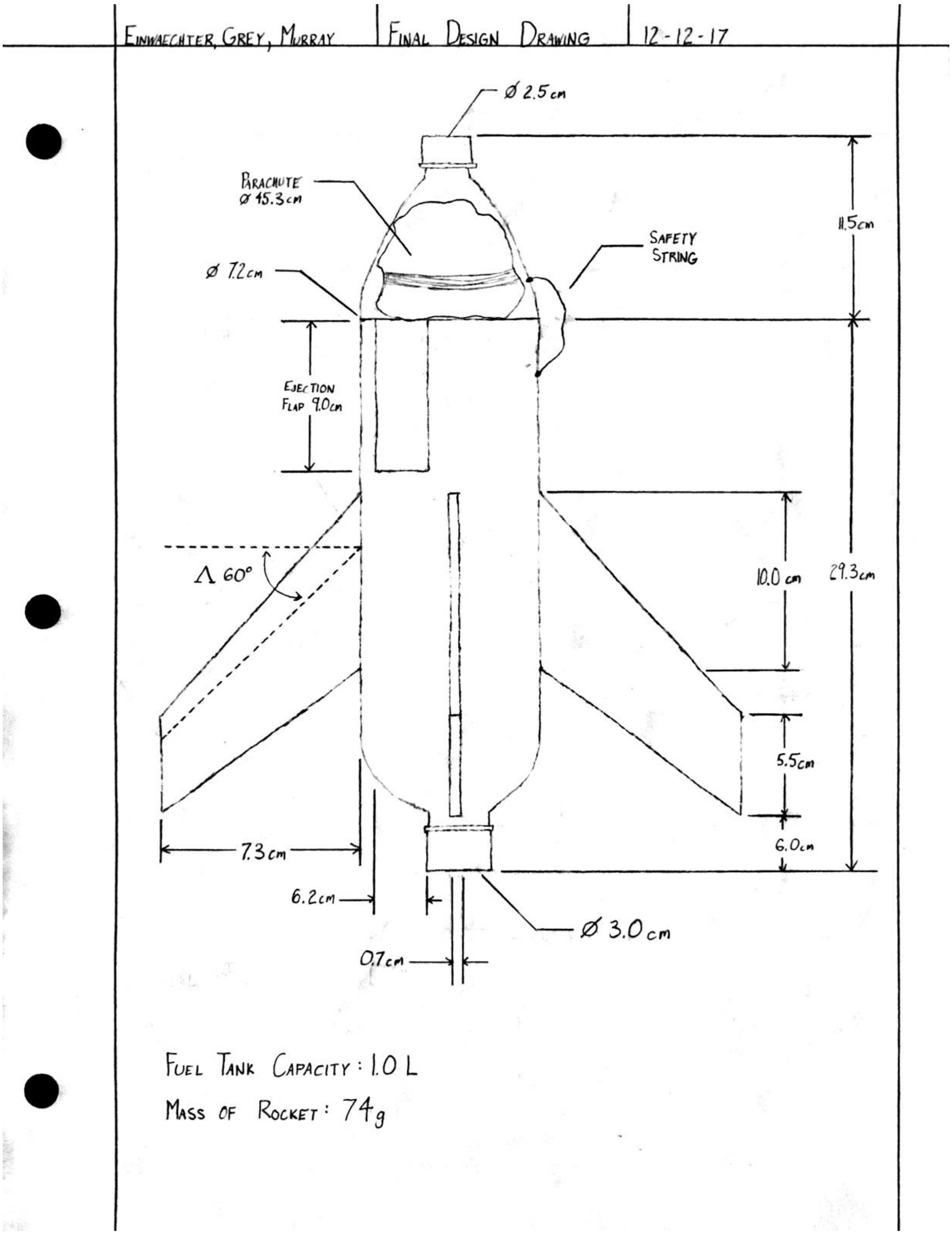

Fig. 2. Completed Dimensioned Drawing of Final Design

Background Theory:

The main forces acting on the rocket during a launch are thrust from the pressure pushing the water out of the nozzle, drag force from the object accelerating through the air, and gravity pushing downward. The formula for the net change in velocity during thrusting can be derived from first considering the basic forces acting on the rocket, gravity and drag. Then the thrust, heavily influenced by exhaust velocity, calculated using Eq. 1., can be derived relative to the speed of the rocket per unit volume. They can then be applied in terms of Bernoulli's equations to calculate the change in velocity per unit volume. When combined, these formulas result in Eq. 2 and can be used calculate the change in velocity over time and project the flight path of a water bottle rocket during the thrust phase.

Finding an equation to model the change in velocity during free flight, after burnout, is much simpler. The rocket is no longer producing thrust by expelling water so the only forces acting on the rocket are gravity and a drag force. By considering Newton's Laws and common formulas for drag force, Eq. 3. can be derived to model the change in velocity during free flight.

$$Exhuast\ Velocity, u = \sqrt{\frac{2P_0}{\rho_w}} * \sqrt{\frac{(\frac{\forall}{\forall_0})^{\gamma} - \frac{P_{atm}}{P_0}}{(1 - \frac{{d_m}^2}{{d_r}^2})^2}}$$

Eq. 1. where P_0 = water pressure, ρ_w = density of water, $\forall$ = current volume of water, $\forall_0$ = starting volume of water, γ = specific heat of air, P_{atm} = atmospheric pressure, d_m = diameter of exhaust nozzle, d_r = diameter of rocket

$$Change\ in\ Velocity\ During\ Thrusting, \Delta V = -g * \Delta t + (\frac{\rho_w * \Delta\forall}{\rho_w * \forall + m_r})u$$

Eq. 2. where g = acceleration due to gravity, Δt = change in time, ρ_w = density of water, $\forall$ = current volume of water, m_r = mass of rocket, u = exhaust velocity calculated in Eq. 1.

$$Change\ in\ Velocity\ During\ Free\ Flight, \Delta V = \left(-g - \frac{(\rho_{air} * v^2 * A_r * C_D)}{2 * m_r}\right) * \Delta t$$

Eq. 3. where g = acceleration due to gravity, Δt = change in time, ρ_{air} = density of air, v = velocity, m_r = mass of rocket, A_r = area of rocket, C_D = drag coefficient

Design Decisions that Use Course Material:

To optimize the performance of the water-bottle rocket design, the variables affecting its maximum velocity must be considered. By examining Eq. 1., Eq. 2., and Eq. 3., derivations which were provided in the course material, initial hypotheses can be made about which variable positively or negatively influence the performance of a water bottle rocket. The variables that increase maximum velocity of the rocket are the water pressure, the volume of water, and the relative velocity. The variables that decrease maximum velocity are gravity, air density, rocket diameter, and coefficient of drag. Air pressure, gravity, and air density are constant. Therefore, the focus of the team is to minimize the diameter and coefficient of drag, while maximize the volume of the water and water pressure. The best combination of these variables was later explored using a computational model created using the equations and process depicted in the course material.

To minimize the coefficient of drag, a paper nose and a plastic nose were discussed to create a more streamlined body. The paper nose was quickly thrown out after learning that paper is not included in the list of materials that can be used. Due to the factors discussed above, a 2L soda bottle and a 1L water bottle were the two “fuel tanks” that were considered. The 2L design had a larger volume, but a smaller diameter than the 1L design. Based on the projected performance of each rocket plotted in MATLAB, the 1L water design was chosen.

Design Decisions that Use Other Resources:

Stability of the rocket is an additional factor that is not considered in the calculations, but must be considered in the design process. Flying a straight path maintains the orientation of the rocket’s streamlined body and in turn, minimizes the drag. To increase the stability of the rocket, several fin designs could be used. The shape of each fin and the number of fins were chosen using an online resource created for students, hobbyists, and teachers (“What is the Best Fin Shape, Size, and Placement”).

The performance of the design is measured by the duration of time from the launch until landing. This means that increasing the maximum velocity of the rocket is not the group’s only concern. From prior experience as model rocket hobbyists, the group knew adding a parachute would increase this duration significantly. A website called US Water Rockets lays out the steps of building water-bottle rockets and briefly discusses each step (“How to Construct an Octagonal Parachute for Your Water Rocket.”). The group loosely used this website for the specific design of its parachute.

Several ideas from a website titled Air Command Water Rockets were discussed by the group to expedite the rocket’s parachute deployment process (“Air Command Water Rockets.”). A simple air flap attached to the nose of the rocket was chosen to separate the nose from the fuel tank. This design is not likely to fail due to its simplicity, and is created using plastic that is cut from the second water bottle.

Initial Performance Estimations:

Initial performance estimates were calculated using a computational model created based around the equations provided in background theory. The first runs were done to ensure realistic results when compared to expected values from research done. Fig. 3 and Fig. 4. show the results from an arbitrary water bottle rocket design. There is a high correlation between the data from our computer model and the models seen in the article “Soda-Bottle Water Rockets” (Source 5). The clear correlation gave confirmation that our model was working properly. The computational model could then be used to analyze the influence of certain design factors on performance and was paramount in final design decisions.

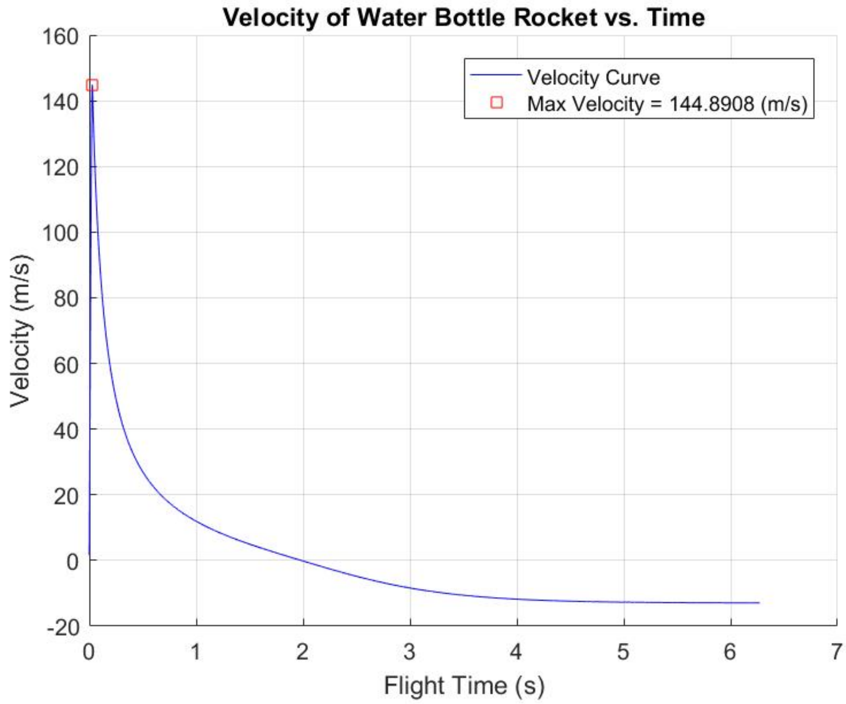

Fig. 3. Initial computational model of velocity over time of a water bottle rocket with: Volume of Bottle = 2 L, Launch Pressure = 60 PSI, Bottle Diameter = 0.110 m, (Volume of Water)/(Volume of Bottle) = 33%, Coefficient of Drag = .75, Empty Mass of Rocket = .074 kg

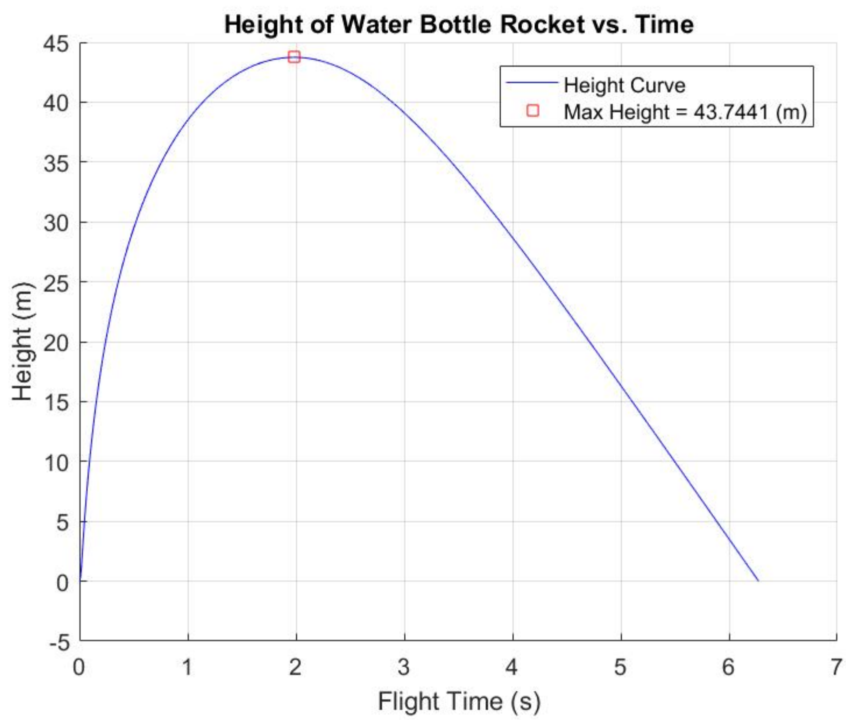

Fig. 4. Initial computational model of altitude over time of a water bottle rocket with: Volume of Bottle = 2 L, Launch Pressure = 60 PSI, Bottle Diameter = 0.110 m, (Volume of Water)/(Volume of Bottle) = 33%, Coefficient of Drag = .75, Empty Mass of Rocket = .074 kg

Design Decisions Based on Computer Model:

For all models, maximum height achieved by rocket was used as performance measurement. The optimized value of the variable is said to be the value which produces the highest altitude with all other variable constant.

The first variable analyzed using the computational model was percent water (Fig. 5.). Testing a range of water from 10% to 40% resulted in a range of maximum altitudes from 40.89 to 47.10 m. The results show that 40% water produced the best performance. This data was the basis for the decision to use 40% water in the final rocket design. The next variable analyzed was the volume of the rocket. The project description limited bottle size to 2L so incremental volumes up to that limit were tested. As anticipated, while holding all other variables constant, specifically the bottle diameter, the best performance was achieved by using the largest volume bottle. This was expected based on course material because having more volume allows more thrust to be produced during ascent. Launch pressure was analyzed next. The maximum launch pressure set by the project description was 60 PSI so pressure at increments of 15 PSI up to this limit were tested. Fig. 7. shows that the best performance was with 60 PSI. This was again expected based on course material. The higher pressure should increase the exhaust velocity, effectively increasing the thrust produced and therefore improving performance.

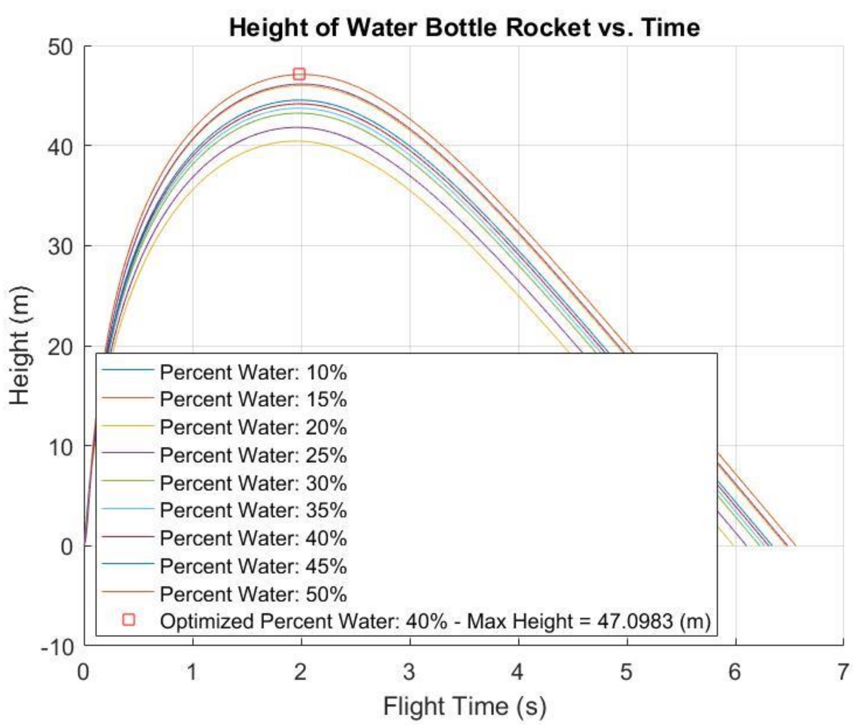

Fig. 5. Exploring optimal percent water, (Volume of Water)/(Volume of Bottle), using computational model of altitude over time of a water bottle rocket with: Volume of Bottle = 2 L, Launch Pressure = 60 PSI, Bottle Diameter = 0.110 m, Coefficient of Drag = .75, Empty Mass of Rocket = .074 kg

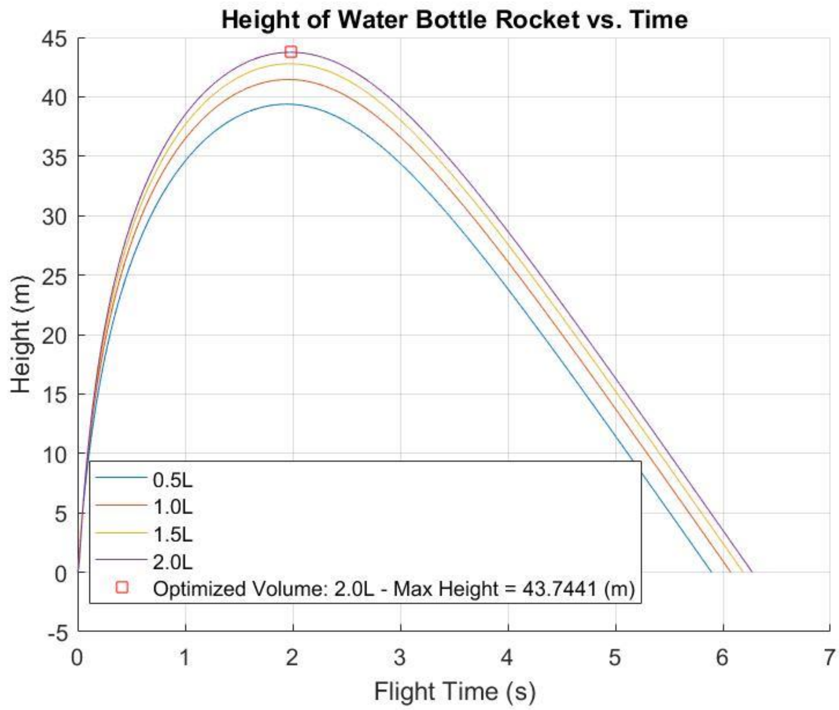

Fig. 6. Exploring optimal bottle volume using computational model of altitude over time of a water bottle rocket with: (Volume of Water)/(Volume of Bottle) = 33%, Launch Pressure = 60 PSI, Bottle Diameter = 0.110 m, Coefficient of Drag = .75, Empty Mass of Rocket = .074 kg

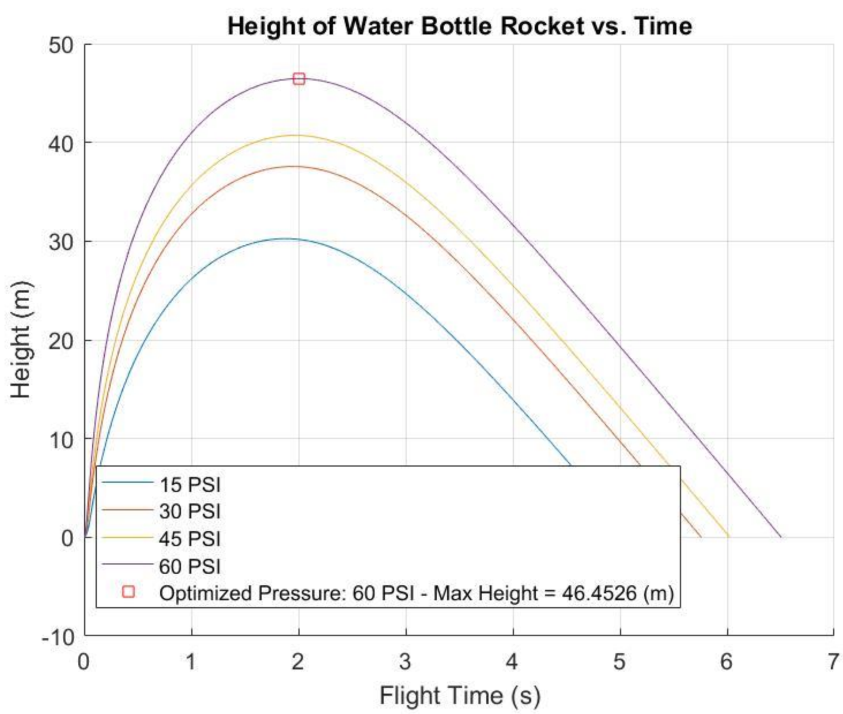

Fig. 7. Exploring optimal launch pressure using computational model of altitude over time of a water bottle rocket with: Volume of Bottle = 2 L, (Volume of Water)/(Volume of Bottle) = 33%, Bottle Diameter = 0.110 m, Coefficient of Drag = .75, Empty Mass of Rocket = .074 kg

The drag force on the rocket during a launch is heavily influenced by the cross-sectional area of a rocket. Based on the material available, either a standard 2L bottle with a cross-sectional diameter of 0.11 m or a smaller 1L bottle with only a 0.072m cross-section diameter could be used. The performance of each of these bottle setups were modeled, producing interesting results. Even though Fig. 6 showed that a higher volume produced a higher altitude, the 1L bottle reached a maximum altitude of 84.3 m compared the 2L which reached 47.1 m. This significant increase in performance is attributed to the area of the cross-section having a higher influence on performance than the volume of the bottle. These results were the sole reasoning behind the design decision to use the 1 L bottle as apposed to the more standard 2 L bottle.

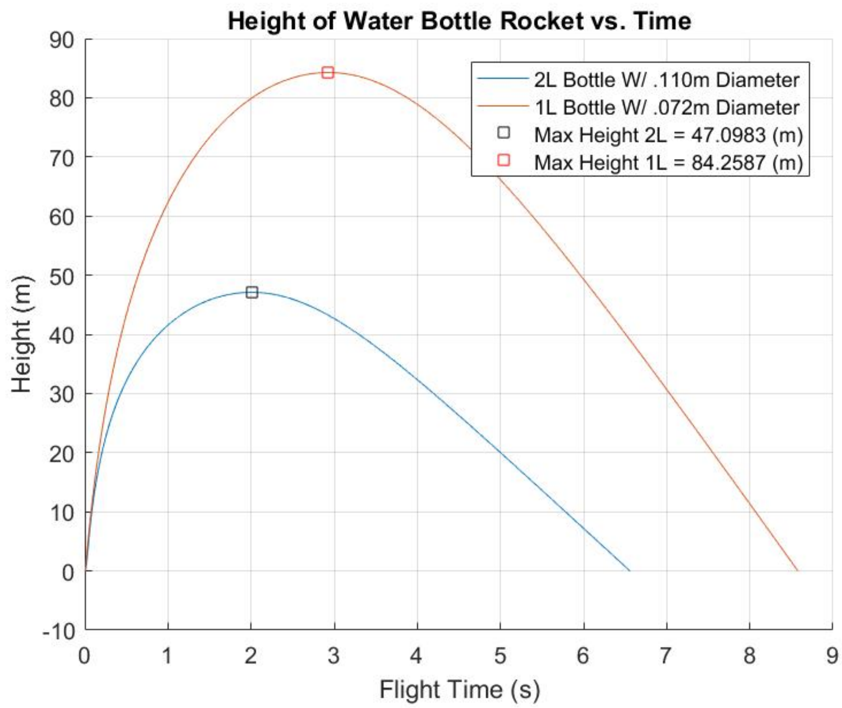

Fig. 8. Comparison of attitude over time of possible bottle volume/diameter combos of a water bottle rocket with: (Volume of Water/(Volume of Bottle) = 40%, Coefficient of Drag = .75, Empty Mass of Rocket = .074 kg

Final Design Solution Summary:

The "fuel tank" used is a 29.3 cm long, 7.2 cm in diameter, 1L water-bottle. A second water bottle was used to create the nose of the rocket. The bottle chosen was comparable in size to the fuel tank. The bottom part was cut of leaving only the 11.5 cm top portion of the bottle to rest on top of the fuel tank. Three fins were cut out of a sturdy, lightweight presentation board. Each fin resembles a swept wing with a 7.3 cm span, 10.0 cm root chord, 5.5 cm tip chord, and a 60-degree sweep angle. The parachute is a circular cutout of a plastic bag that is 45.3 cm in diameter. The plastic air flap is 9.0 cm in length and 6.2 cm in width. The design is assembled using duct tape. The assembled design's mass is 74 grams. Final launch specifications were chosen to be 400 mL of water (40% water) and 60 PSI based on computation models. A dimensioned diagram of the final design is provided in Fig. 2. and an image of the actual rocket is in Fig. 1.

Final Performance Estimates:

The computation model was run using the specifications of the final rocket design to calculate the expected maximum altitude. Unlike other runs, for the final performance estimate a correction factor of .8 was added to both the height and velocity calculations. The correction factor was added due to research done of maximum altitudes of water bottle rockets and anticipated optimism in the equations used in the model. The equations are optimistic because they do not account for factor such as wind, leaking of pressure immediately before launch, and other factors which would negatively influence the performance. After taking the correction factor into account, the expected maximum altitude is 64.61 m, 2.9284 seconds after launch. The expected maximum velocity of the rocket is 113.9 m/s, just 0.0164 seconds after launch.

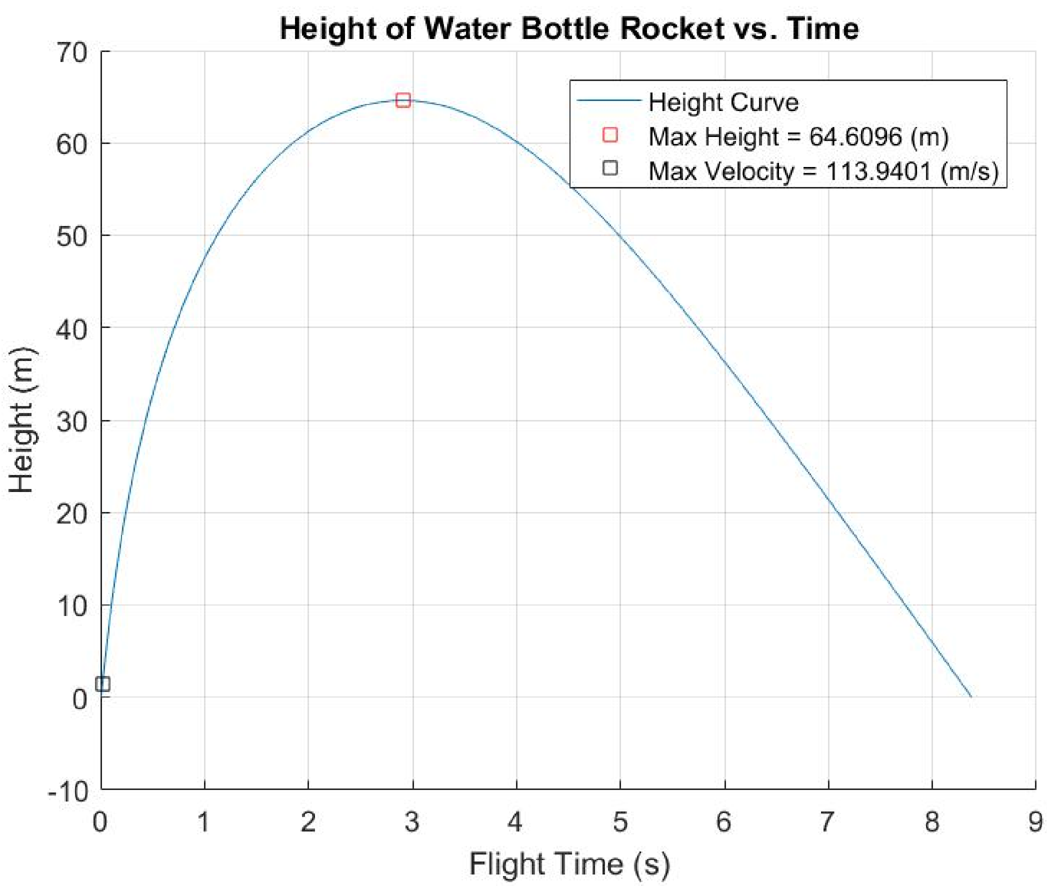

Fig. 9. Final computer model for height over time of final water bottle rocket with: Volume of Bottle = 1 L, Launch Pressure = 60 PSI, Bottle Diameter = 0.072 m, (Volume of Water)/(Volume of Bottle) = 4%, Coefficient of Drag = .75, Empty Mass of Rocket = .074 kg, using a correction factor of .8 for both height and velocity

Sources

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2. Edwards, Jack “Lecture 10.” Introduction to Aerospace Vehicle Performance. North Carolina State University, Raleigh NC. 1 Dec. 2017
3. “How to Construct an Octagonal Parachute for Your Water Rocket.” *US Water Rockets*, 5 July 2003, www.uswaterrockets.com/construction_&_tutorials/Parachute/tutorial.htm.
4. “What Is the Best Fin Shape, Size, and Placement?” *Water-Rockets Science*, 11 Dec. 2009, www.water-rockets.com/article.pl?121%2C0.
5. The Physics Teacher 33, 150 (1995); doi: 10.1119/1.2344175 http://dx.doi.org/10.1119/1.2344175

Matlab Code:

```
function [time_data,height_data,velocity_data] = WaterRocketLaunch( vol_bottle, percent_water, p, C_d,
mode, bottle_dia )
% Water Bottle Rocket Project
% MAE 250, North Carolina State University
% Sean Murray, Conner Grey, James Einwaechter
% 12/12/2017
% Description:
%   Takes initial variables of a water bottle rocket and models it's height
%   and velocity over time
% Inputs:
%   p: Rocket Pressure, (N/m^2)
%   percent_water: (volume of water in rocket)/(volume of rocket)
%   vol_bottle: Volume of rocket, (m^3)
%   mode: 0 - model boosted phase
%         1 - model boosted and free flight phases
%         2 - model to peak altidude
%   bottle_dia: Diamater of Water Bottle Rocket (m)
% Outputs:
%   time_data: matrix with time of each loop iteration
%   height_data: matrix with height of each loop iteration
%   velocity_data: matrix with velocity of each loop iteration

% Initilize Variables, SI Units

t = 0;  % Time, (s)
v = 0;  % Velocity, (m/s)
h = 0;  % Height, (m)
vol_water = vol_bottle*percent_water; % Volume of Water, (m^3)
vol_air = vol_bottle - vol_water; % Volume of Air, (m^3)
delta_t = 0; % Change in Time, (s)
delta_vol = 1e-5; % Change in Volume, (m^3)
time_data = [];
height_data = [];
velocity_data = [];

% Initilize Constants, SI Units

p_atm = 1.0018e5;   % Pressure, (N/m^2)
g = 9.8064; % Gravity, (m/s^2)
rho_water = 1000;   % Density of Water, (kg/m^3)
rho_air = 1.2137; % Density of Air, (kg/m^3)
rocket_mass = .074; % Mass of Rocket, (kg)
d_hole = .0215; % Diameter of Hole, (m)
d_rocket = bottle_dia; % Diameter of Rocket, (m)
initial_vol_water = vol_water; % Initial Volume of Air (m^3))

% Loop Until Burnout
while (vol_water >= 0)
```

```
    u = sqrt(2*p/rho_water)*sqrt(((vol_water/(initial_vol_water)^1.4)-p_atm/p)/(1-
((d_hole^2)/(d_rocket^2)))^2);
    delta_t = delta_vol/(u*pi/4*d_hole^2);
    t = t + delta_t;
    vol_air = vol_air + delta_vol;
    vol_water = vol_water - delta_vol;
    delta_v = -g*delta_t+(rho_water*delta_vol)/(rho_water*vol_water+rocket_mass)*u;
    v = v + delta_v;
    h = h + v*delta_t;

    time_data = [time_data,real(t)];
    height_data = [height_data,real(h)];
    velocity_data = [velocity_data,real(v)];
end

if mode == 1
    % Loop Until Landing
    delta_t = .001;

    while (h >= 0)
        if v > 0    % Rocket Climing
            delta_v = (-g-.5*rho_air*v^2*(pi*(d_rocket^2)/4)*C_d/rocket_mass)*delta_t;
        else    % Rocket Falling
            delta_v = (-g+.5*rho_air*v^2*(pi*(d_rocket^2)/4)*C_d/rocket_mass)*delta_t;
        end
        v = v + delta_v;
        h = h + v*delta_t;
        t = t + delta_t;
        time_data = [time_data,real(t)];
        height_data = [height_data,real(h)];
        velocity_data = [velocity_data,real(v)];
    end
elseif mode == 2
    % Loop Until Max Height
    delta_t = .001;

    while (v >= 0)
        if v > 0    % Rocket Climing
            delta_v = (-g-.5*rho_air*v^2*(pi*(d_rocket^2)/4)*C_d/rocket_mass)*delta_t;
        else    % Rocket Falling
            delta_v = (-g+.5*rho_air*v^2*(pi*(d_rocket^2)/4)*C_d/rocket_mass)*delta_t;
        end
        v = v + delta_v;
        h = h + v*delta_t;
        t = t + delta_t;
        time_data = [time_data,real(t)];
        height_data = [height_data,real(h)];
        velocity_data = [velocity_data,real(v)];
    end
end
end
```